EECS 182 Deep Neural Networks
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Discussion 9

1. Setting up sequence prediction as RNN

The idea of recurrent information processing is quite basic and predates deep neural networks. Consider the following formulation:

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{u}_t \tag{1}$$
$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + \mathbf{w}_t \tag{2}$$

This recurrent form has $\mathbf{x}_t$ denoting the (soon to be hidden) state at timestep t, $\mathbf{y}_t$ is the measurement (or label), $\mathbf{u}_t$ is some driving noise (assume it to be zero-mean iid Gaussian) at timestep t, and $\mathbf{w}_t$ is the similarly zero-mean iid Gaussian observation noise at each timestep. Here $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are the weights that determine the state evolution and observations at **all** time steps.

(a) In the case where all weights $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are just matrices and we have access to full trajectories of $(\mathbf{x}_t, \mathbf{u}_t, \mathbf{y}_t)$, **how can you setup the problem of learning $\mathbf{A}, \mathbf{B}, \mathbf{C}$ from this training data?**

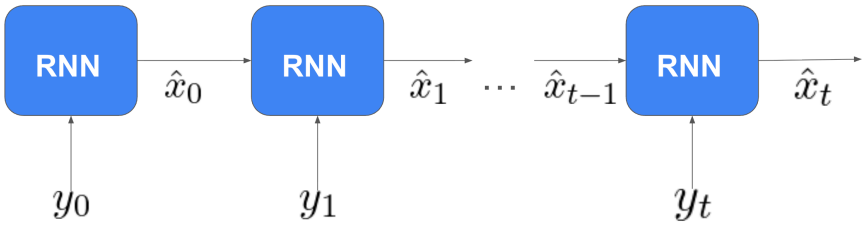
(b) Assume that the $\mathbf{u}_t = \mathbf{0}$ for simplicity. We know from Kalman filtering that if we know the $\mathbf{A}, \mathbf{B}, \mathbf{C}$ matrices, it is possible to create a steady-state filter which approximately tracks that state $\mathbf{x}_t$ from (1) given only the sequence of observations $\mathbf{y}_t$ as long as the (A, C) matrix is observable — note "observability" here is a technical condition that is dual to "controllability" and says that $[C, CA, CA^2, \ldots]$ is full rank. Such a filter can be given in recursive form as:

$$\widehat{\mathbf{x}}_t = \mathbf{A}'\widehat{\mathbf{x}}_{t-1} + \mathbf{B}'\mathbf{y}_t \tag{3}$$

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Now, suppose we actually had training-time access to full trajectories of $(\mathbf{x}_t, \mathbf{y}_t)$ and want to learn $\mathbf{A}', \mathbf{B}'$ as in (3). **Comment on why simply using the approach of the previous part does not work and we can't use the ground-truth $\mathbf{x}_t$ to break the long chain of dependency in the state evolution of the filter across the sequence in the way that we were able to do for simple system identification with perfectly observed states.**

(c) Continuing the previous part (continue to assume that the $\mathbf{u}_t = \mathbf{0}$ for simplicity), **how can you train on full trajectories of $(\mathbf{x}_t, \mathbf{y}_t)$ in order to learn $\mathbf{A}', \mathbf{B}'$ from (3)?** Also, **modify the computation graph below to show what extra computations are required to compute the loss function.**

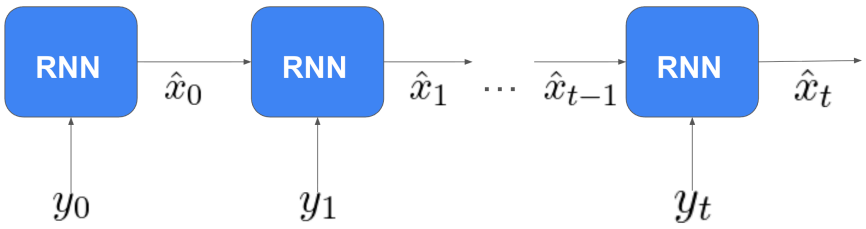


Note: in this case, for simplicity, do not attempt to setup any loss function involving trying to reconstruct the $\mathbf{y}_t$. Simply treat these as known inputs for this part.

While the filter doesn't see $\mathbf{x}_t$ at training time, we can define a loss function on how close $\widehat{\mathbf{x}}_t$ is to $\mathbf{x}_t$, and optimize the loss function to learn $\mathbf{A}'$ and $\mathbf{B}'$.

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(d) Now consider the case where we only have measurements of $\mathbf{y}_t$ for all time steps. (Continue to assume that the $\mathbf{u}_t = \mathbf{0}$ for simplicity.) This is common in applications such as **object tracking**, where you only have a series of video frames, and the model has to track the movement of an object throughout the frames. **First, show how an RNN model can be setup to predict the next $\mathbf{y}_{t+1}$ in the sequence and how this RNN can be trained.** Afterwards, **modify the same computation graph to include the new loss function.**



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(e) To simplify computation, now assume that $\mathbf{w}_t = \mathbf{0}$. Given that the sequence length is T (input: $(\mathbf{u}_1, \cdots, \mathbf{u}_T)$, output: $(\mathbf{y}_1, \cdots, \mathbf{y}_T)$) and assuming the initial state $\mathbf{x}_0 = 0$, **show that the output $\mathbf{y}_t$ can be expressed as a *convolution* of the input sequence $\{\mathbf{u}_i\}_1^T$ with a kernel $K = \{K_i\}_0^T$**:

$$\mathbf{y}_t = \sum_{i=0}^{T} K_{t-i}\mathbf{u}_i,$$

where any K_j for $j < 0$ is set to $\mathbf{0}$ to ensure causality. **Find K.**

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